Hong Kong Scholars Symposium 2021

Topology Optimization of Viscoelastically Damped Structures Under Time-Dependent Loadings:

Sensitivity Analyses and Computational Considerations

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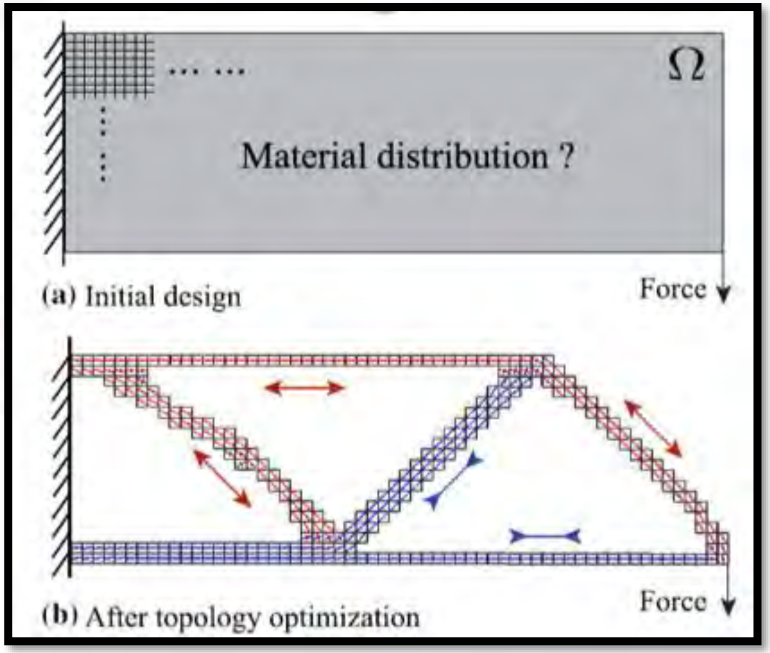
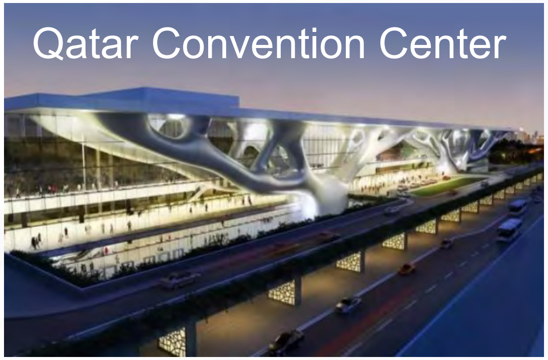
Outline

- Backgrounds
- Motivations and Difficulties
- Current Work and Results
- Conclusions and Outlook
- Acknowlegements

Topology Optimization



- ✓ find the optimal layout of a structure within a given region under a set of loads, boundary conditions and constraints
- ✓ Widely applied in conceptual design
- ✓ Manufacturability → Additive manufacturing
- ✓ Undamped/ Viscously damped



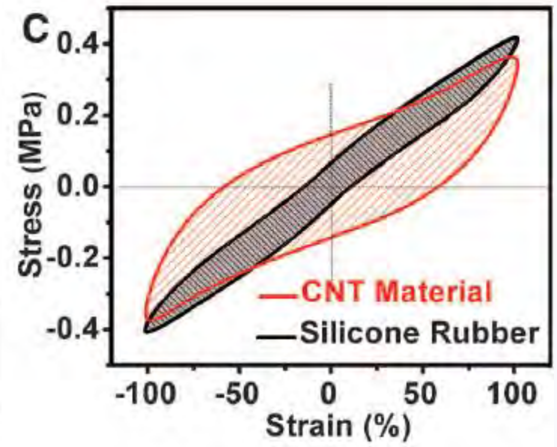
Architecture Design **3D Printing** **Auxetic Structures**

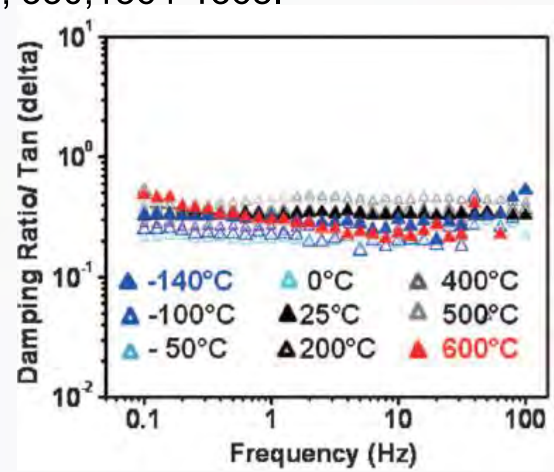
http://www.xieym.com/

Struct. Multidisc. Optim 2018, 57(6): 2457–2483.

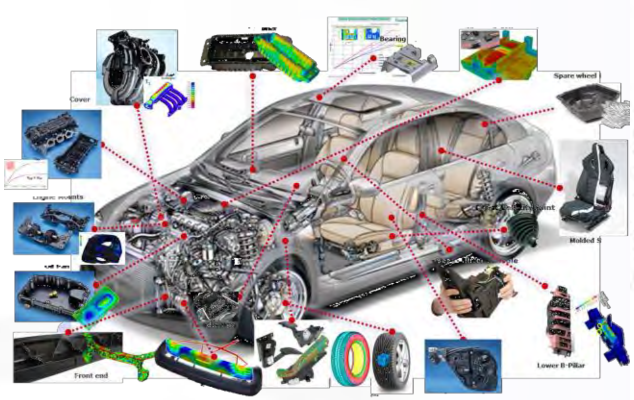
Viscoelastic Materials

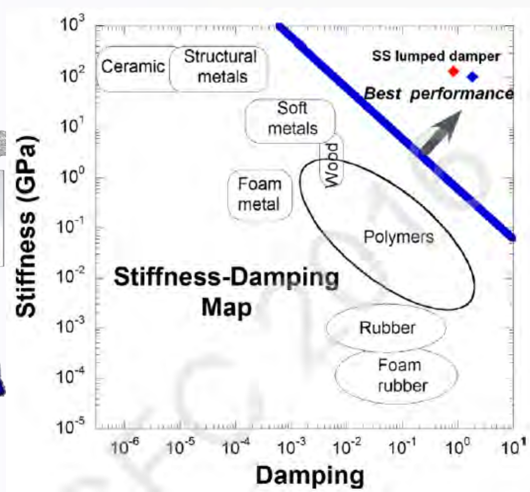
Science, 2010, 330,1364-1368.





- ✓ both elasticity and viscosity
- ✓ Wider range of frequency and temperature





INT J SOLIDS STRUCT, 2013, 50(14-15): 2416-2423.

Non-viscous Damping

Def : not proportional to the instantaneous velocity

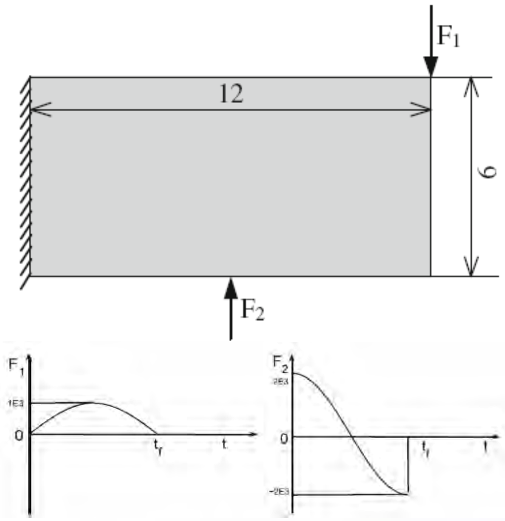
Convolution integral form:

$$f_d(t) = -\int_{-\infty}^{t} g(t-\tau)\dot{x}(\tau)\,\mathrm{d}\tau$$

Most general damping model within the scope of linear system

Motivations

Struct. Multidisc. Optim 2016, 53(1): 101–114.



- DOF: 60 X 30
- Undamped
- 100 steps
- Newmark method
- 65.4 X 311 = 20336 s

Iteration 5.64 h

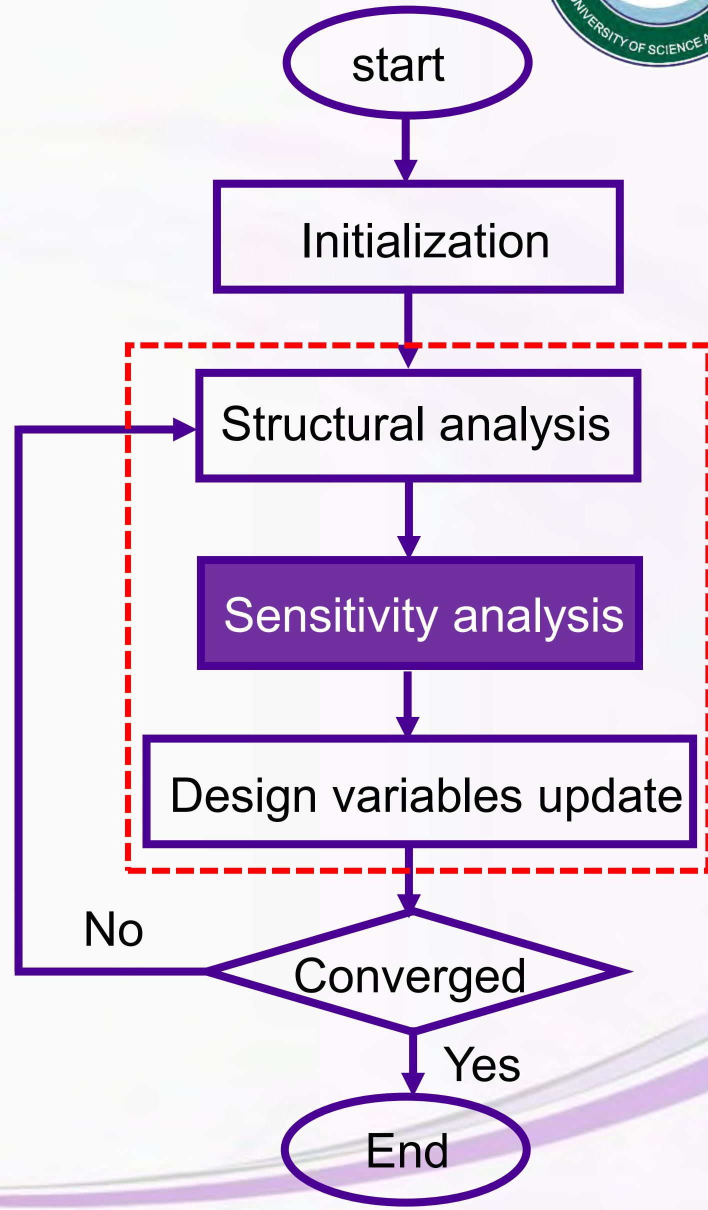
✓ Increased dimensions
✓ Complex modes

↓

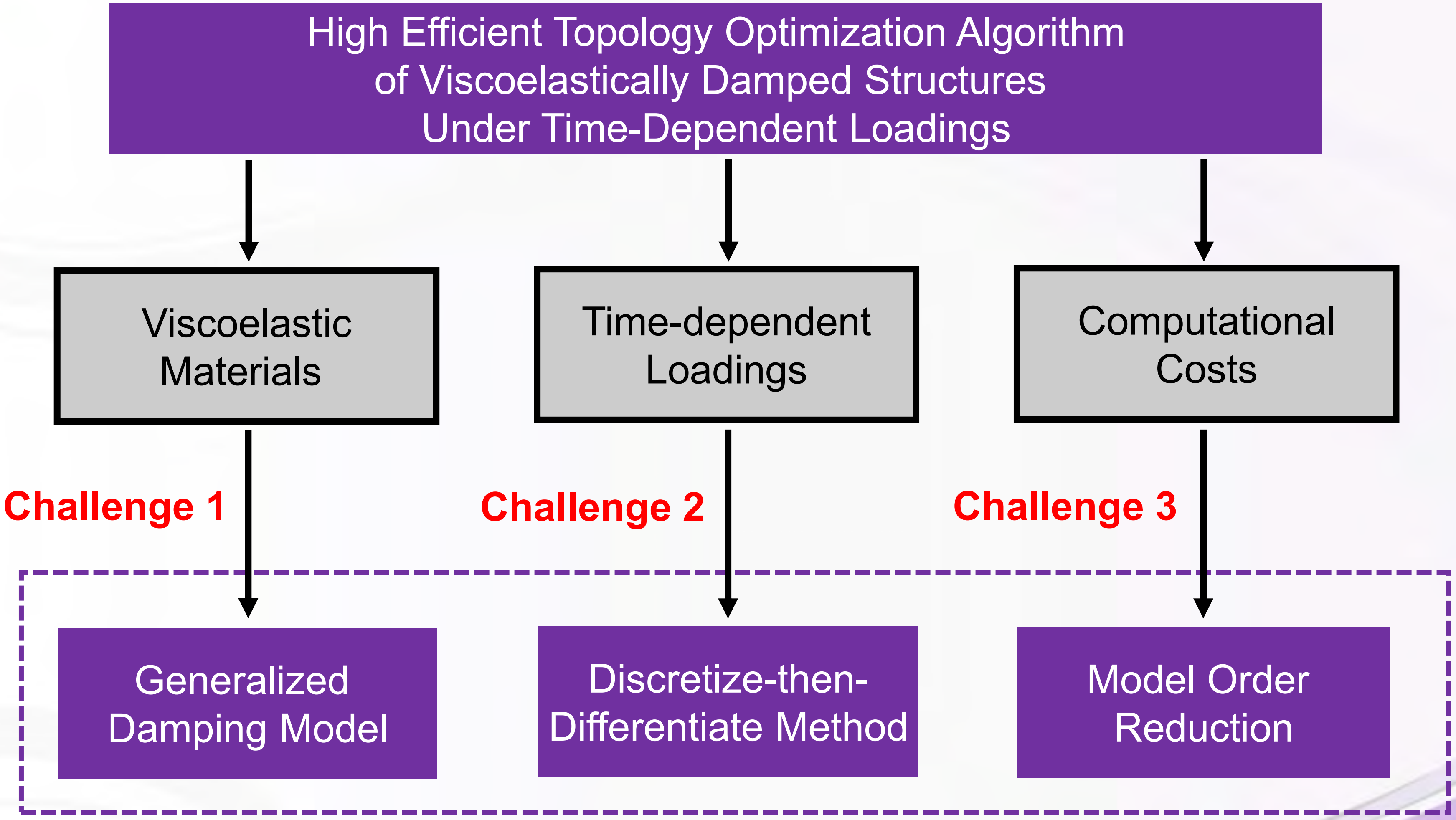
□ Efficient TO framework

↓

✓ Time integration method
✓ TO method/ optimization algorithm
✓ Computational considerations



Difficulties



Challenge 1

- **Transient responses for viscoelastic systems**
  - Generality
  - Accuracy and efficiency

Mathematical representations of some non-viscous damping models.

Non-viscous damping models	Kernel functions
Biot model	$\mathbf{G}(s)=\sum_{k=1}^{n}\frac{a_k s}{s+b_k}\mathbf{C}_k$
Exponential damping model	$\mathbf{G}(s)=\sum_{k=1}^{n}\frac{c\mu_k}{s+\mu_k}\mathbf{C}_k$
Golla-Hughes-McTavish (GHM) model	$s\mathbf{G}(s)=G_\infty\left[1+\sum_{k=1}^{n}\alpha_k\frac{s^2+2\xi_k\omega_k s}{s^2+2\xi_k\omega_k s+\omega_k^2}\right]\mathbf{C}_k$
Anelastic Displacement Field (ADF) model	$\mathbf{G}(s)=\sum_{k=1}^{n}\frac{\Delta_k}{s+\Delta_k}\mathbf{C}_k$
Fractional derivative model	$\mathbf{G}(s)=\frac{E_1 s^\alpha-E_0 b s^\beta}{1+bs^\beta}\mathbf{C}(0<\alpha,\beta<1)$
Step-function model	$\mathbf{G}(s)=c\frac{1-e^{-st_0}}{st_0}\mathbf{C}$
Half cosine model	$\mathbf{G}(s)=\frac{c}{st_0}\frac{1+2(st_0/\pi)-e^{-st_0}}{1+2(st_0/\pi)^2}\mathbf{C}$
Gaussian model	$\mathbf{G}(s)=ce^{s^2/4\mu}\left[1-\mathrm{erf}\left(\frac{s}{2\sqrt{\mu}}\right)\right]\mathbf{C}$

How to develop a **general** and **easy-to-solve** mathematical expression to make the method applicable to **the majority of existing damping models** ?

Methodology

➢ Generalized Damping Model (GDM)

$$G_k(s) = \frac{c_{p_k} s^{p_k} + c_{p_{k-1}} s^{p_{k-1}} + \cdots + c_0}{d_{q_k} s^{q_k} + d_{q_{k-1}} s^{q_{k-1}} + \cdots + d_0}$$

- $p_k = q_k = 0$ ⟶ **Viscous damping model**
- $p_k = q_k = 1$ ⟶ **BIOT model & ADF model**
- $p_k = q_k = 2$ ⟶ **GHM damping model**

✓ **Mathematically expressed by a fraction formula**

✓ **Introduce a unified way to express the frequently-used viscous/non-viscous damping models.**

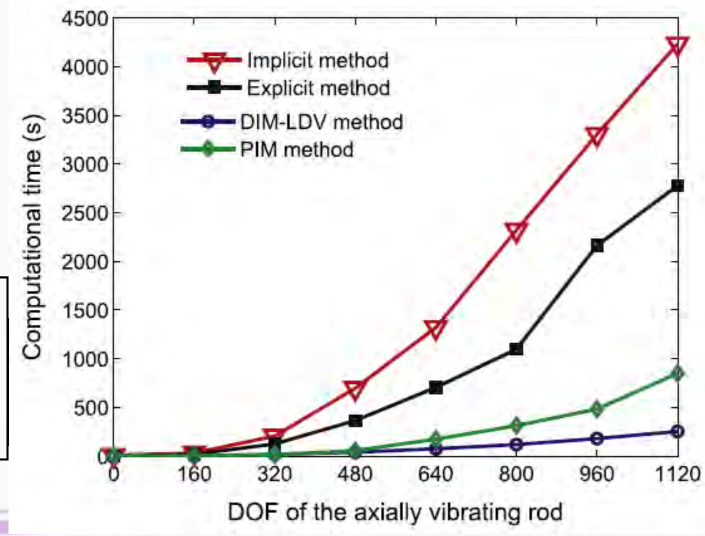
➢ Modified Precise Integration Method (MPIM)

Equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}_0\dot{\mathbf{x}}(t) + \sum_{k=1}^{n} \mathbf{C}_k \int_0^t g_k(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t)$$

State-space formulation:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{r}(t),\ \ where\ \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_N & \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_0 & -\mathbf{M}^{-1}\mathbf{L} \\ \mathbf{0} & -\mathbf{W}^{-1}\mathbf{R}^T & \mathbf{W}^{-1}\mathbf{E} \end{bmatrix}$$



Z. Ding, et al., *Mech. Syst. Signal Pr.* 2019, 121: 322-342.

Challenge 2

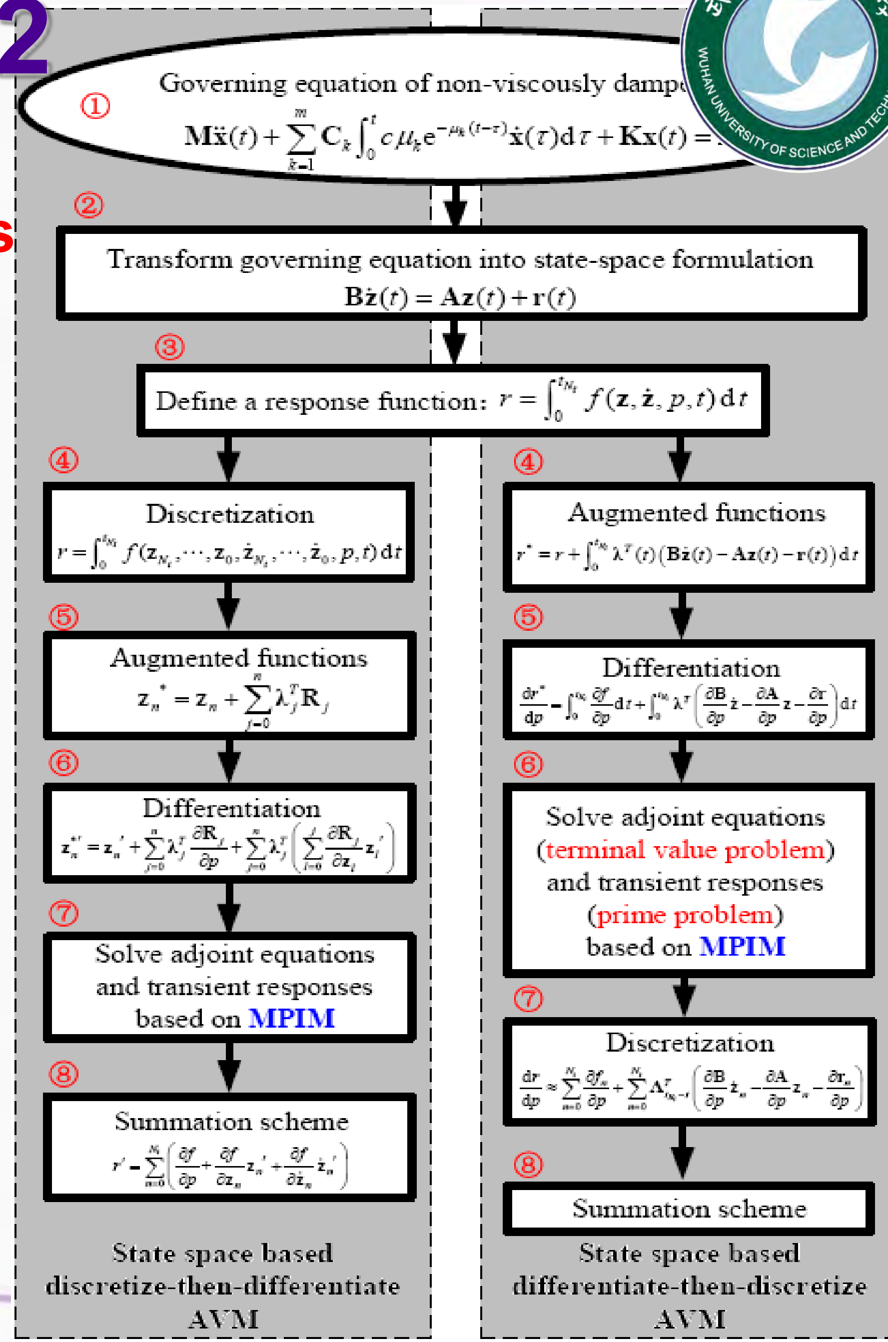
➢ Computational Considerations

- Accuracy
- Consistency
- Implementation effort
- Computational complexity

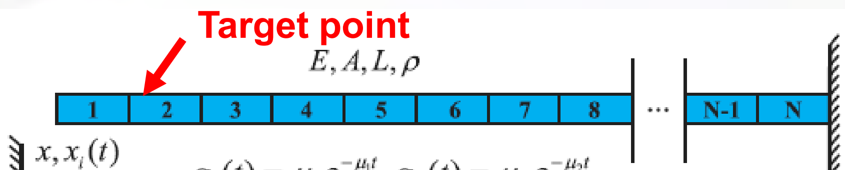
➢ Factors

- the approximations of damping forces
- time integration method
- time step size
- summation scheme
- order of discretization and differentiation

When using MPIM, unlike the undamped systems, the order of discretize and differentiation has no obvious effects on reducing the inconsistency.



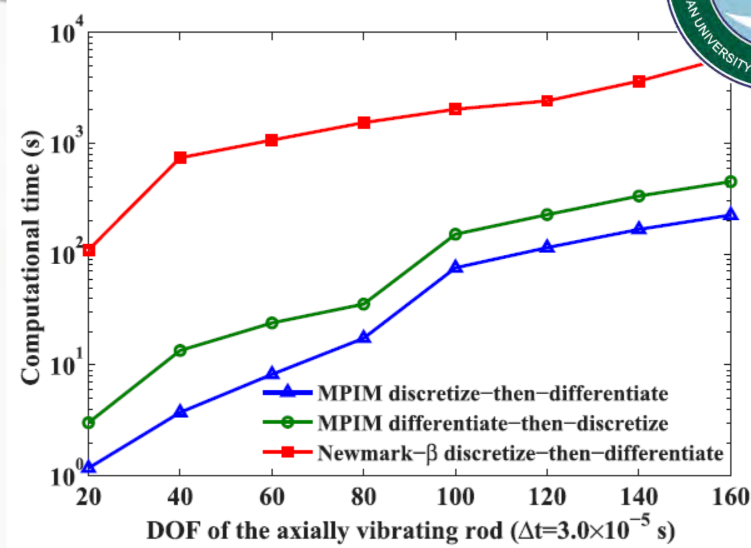
Case study



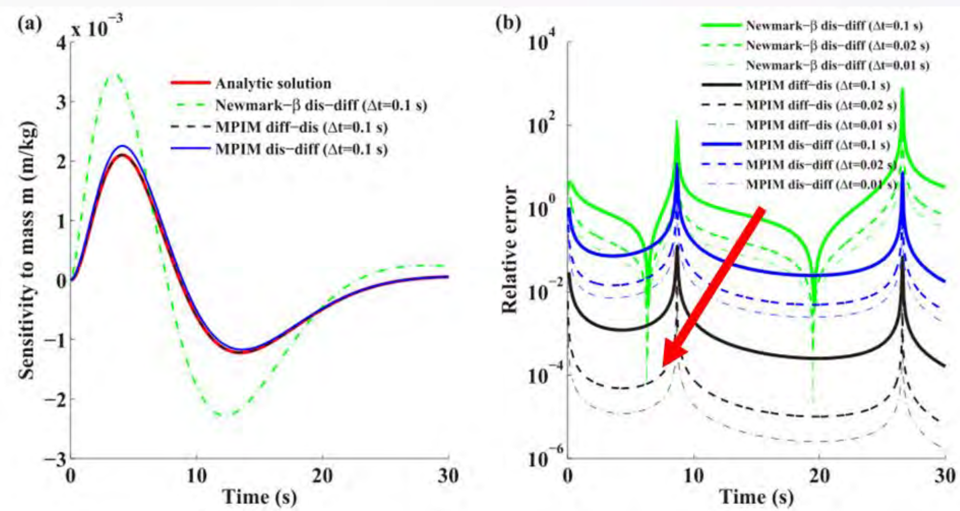
$$g_1(t)=\mu_1\,\mathrm{e}^{-\mu_1 t},\ g_2(t)=\mu_2\,\mathrm{e}^{-\mu_2 t}$$

Items	Values & Expressions
Rod length (L)	4 m
Youngs modulus (E)	2.1×10^{11} N/m^2
Density (ρ)	7.8×10^{3} kg/m^3
Cross sectional area (A)	6.25×10^{-4} m^2
Damping factor (ξ)	0.05
γ_1	1
γ_2	2
Rod element length (l_e)	L/N
1st-order natural frequency (ω_1)	$\sqrt{\frac{E}{\rho}}\frac{1}{2L}\pi$
2nd-order natural frequency (ω_2)	$\sqrt{\frac{E}{\rho}}\frac{3}{2L}\pi$
Highest natural frequency ($\omega_{\max}$)	$\sqrt{\frac{E}{\rho}}\frac{2N-1}{2L}\pi$
Lowest time period ($T_{\min}$)	$\frac{2\pi}{\omega_{\max}}$
Relaxation parameter μ_1	$\frac{1}{\gamma_1 T_{\min}}$
Relaxation parameter μ_2	$\frac{1}{\gamma_2 T_{\min}}$

- **N is changeable**
- **Free vibration**
- **Initial conditions** : $\mathbf{x}_0=\mathbf{0},\dot{\mathbf{x}}_0=\{1,0,\cdots,0\}^T$

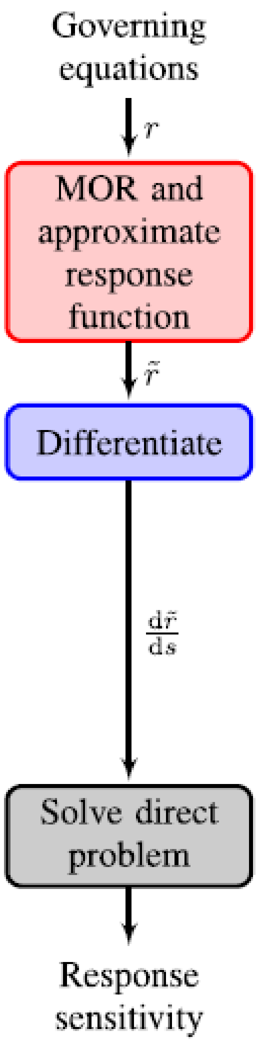


- **Efficiency:** Dis-diff > diff-dis
- **Accuracy:** Diff-dis > dis-diff



Z. Ding, et al., *Struct. Multidisc. Optim* 2021, https://doi.org/10.1007/s00158-021-02937-9

Challenge 3



➢ Model Order Reduction (MOR) Techniques

- **Normal Mode based**
  - ✓ High computational efficiency
  - ✗ Low approximation accuracy (iterative based method)
- **Complex Mode based**
  - ✓ High accuracy for highly damped systems
  - ✗ Computational inefficiency (obtain complex modes)

Which projection basis yields the best trade-off between efficiency and accuracy on computing the sensitivity of transient responses for viscoelastically systems?

Projection bases

- **Multi-model (MM) Method**

  -- Combine several modal bases

  Static correction | Normal modes

  $$\mathbf{T}_{MM} = \left[\mathbf{X}_{cor}, \mathbf{T}_{p_1}, \cdots, \mathbf{T}_{p_m}\right].$$

- **Modal Strain Energy Modified by Displacement Residuals (MSER)**

  -- iteratively seeking a better approximation

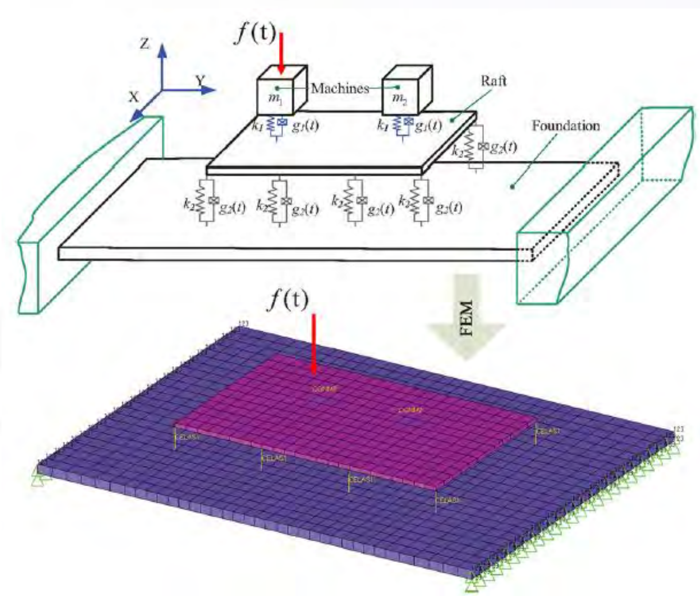
  $$\mathbf{T}_{MSER} = \left[\mathbf{T}_{MSE}, \mathbf{R}_d^*\right].$$

  Normal modes | Displacement residuals

- **Improved Approximation Method (IAM)**

  -- Complex modes based

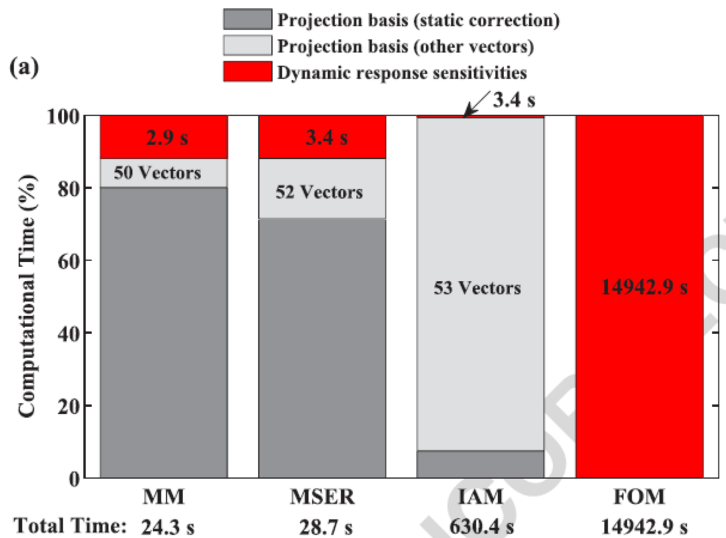
  $$\mathbf{T}_3 = \sum_{j=1}^{L} \frac{\varphi_j^T \mathbf{F}(s) \varphi_j}{\lambda_j^2 \theta_j} - \mathbf{K}^{-1}\mathbf{G}_0\mathbf{K}^{-1}\mathbf{F}(s)$$

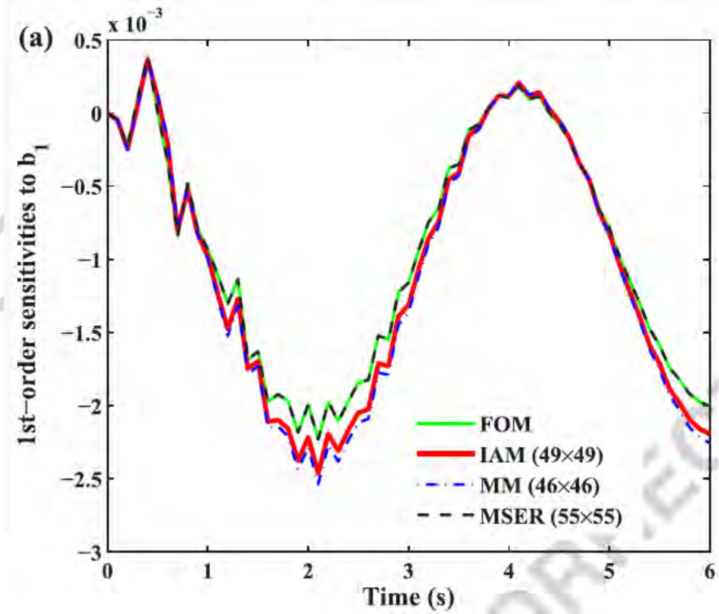


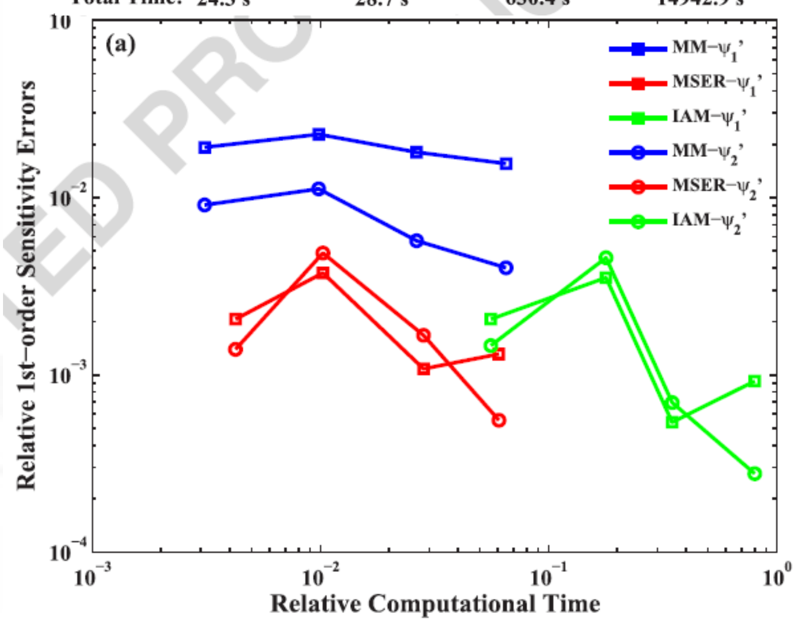
Items	Values
Mass (m_1)	200 kg
Mass (m_2)	250 kg
Raft (length-width-thickness)	1200-800-20 mm
Foudation (length-width-thickness)	2000-1600-40 mm
Young's modulus (E)	2.1×10^{11} N/m^2
Density (ρ)	7.8×10^{3} kg/m^3
Stiffness (k_1)	1.0×10^{5} N/m
Stiffness (k_2)	5.0×10^{5} N/m
Biot damping coefficient (a_0)	1.4970×10^{-2}
Biot damping coefficient (a_1)	2.0132 × 10^[illegible]
Biot damping coefficient (b_1)	5.5893
Exponential damping coefficient (c_1)	10
Exponential damping coefficient (μ_1)	10

- DOFs: 7202 (+120)
- g1(t) : BIOT
- g2(t) : exponential model
- f(t)=5sin(0.5pi$t$)

Case study





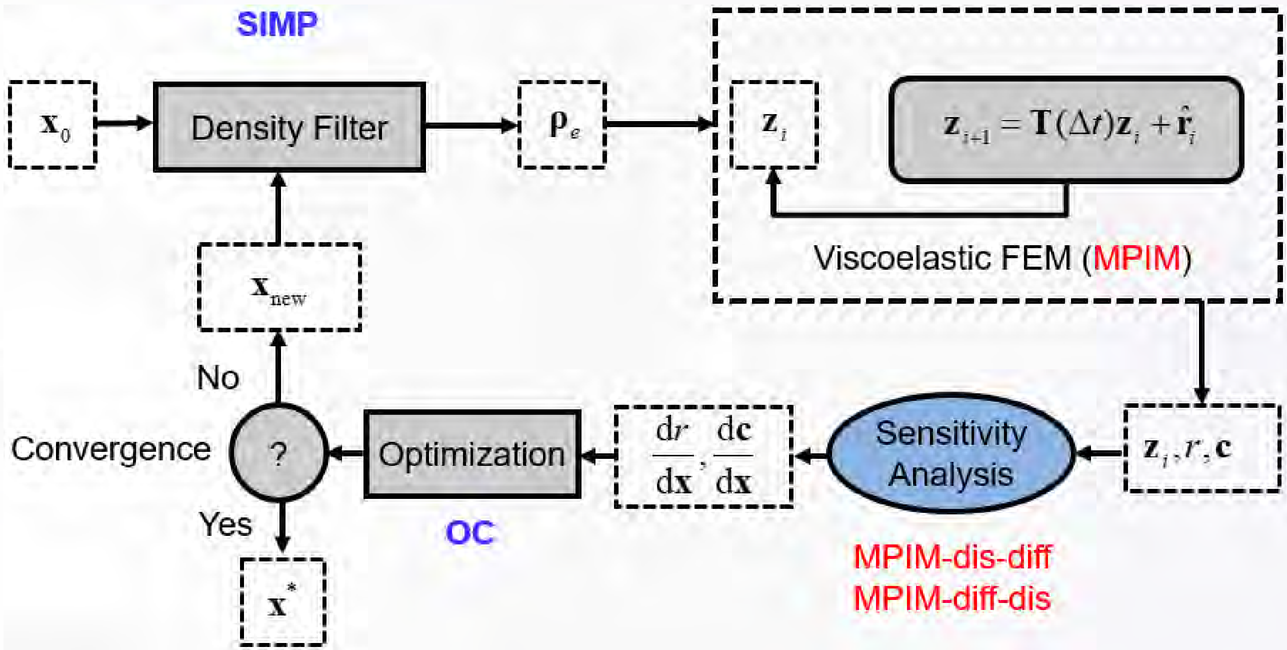


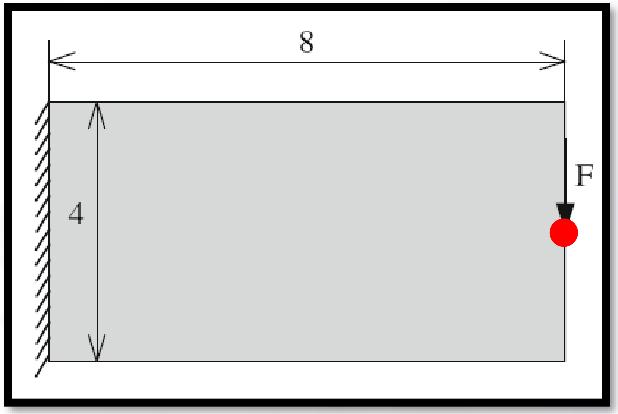
Conclusions：

- FOM is very time-consuming, ROMs significantly reduce the computational time
- Complex modes are less efficient
- MSER yields best trade-off, but is less stable than MM
- Recommend MM

Z. Ding, et al., *Struct. Multidisc. Optim* 2021, https://doi.org/10.1007/s00158-021-02937-9

TO Framework





Design domain and boundary condition

Mean Strain Energy:

$$r\left(\mathbf{u}(t), \rho_e\right) = \frac{1}{2}\mathbf{u}^T(t)\mathbf{K}\left(\rho_e\right)\mathbf{u}(t)$$

Volume Constraint: $V_{\max} = 0.5$

Force Cycle: $F(t) = 1000\sin(\pi t / t_f),\ t_f = 0.05$ and 0.02

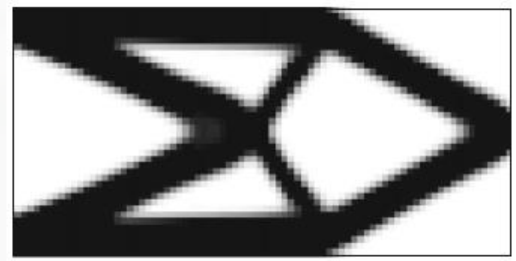
Mesh Size: 80 X 40

Clamped Plate/2D

Mean Strain Energy:

$$r\left(\mathbf{u}(t),\rho_e\right)=\frac{1}{2}\mathbf{u}^T(t)\mathbf{K}(\rho_e)\mathbf{u}(t)$$

Time Step Size: 0.001



0.00837
0.00960

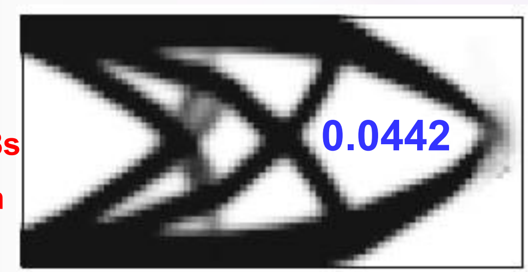
Static Design

82 X
123.58s
≈2.81h

MPIM-dis-diff, t_f=0.05

78 X
150.78s
≈3.27h

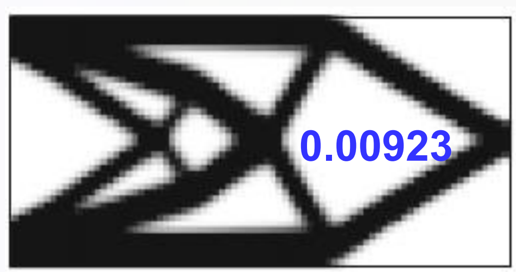
MPIM-diff-diff, t_f=0.05



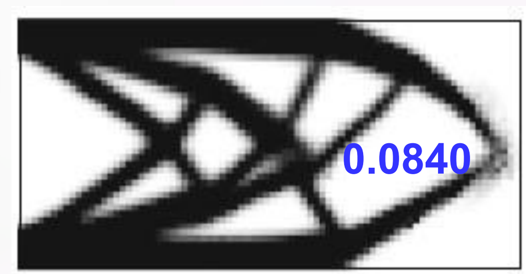
87 X
400.30s
≈9.67h

Newmark-dis-diff, t_f=0.05

MPIM-dis-diff, t_f=0.03



MPIM-diff-dis, t_f=0.03



Newmark-dis-diff, t_f=0.03

Conclusions & Outlook

Conclusions:

1. Generalized Damping Model (GDM): a unified approach for modeling viscoelastically damped systems.
2. The order of discretization and differentiation has no obvious effect on calculating the transient responses of viscoelastic systems and the state-space based discretize-then-differentiate method is recommended.
3. Multi-model (MM) is more suitable than other compared projection bases for reducing the dimension of viscoelastic systems.
4. There is a great desire to improve the computational efficiency of TO. The computational time can be significantly decreased by efficient sensitivity analysis method.

Outlook:

1. Incorporating the MOR into the TO
2. Topology design of the viscoelastic damping material core for sandwich structures under time dependent loadings and experimental validations.

Publications

1. **Zhe Ding**, Lei Zhang, Qiang Gao, Wei-Hsin Liao*. State-space based discretize-then-differentiate adjoint sensitivity method for transient responses of non-viscously damped systems. *Computers & Structures*, 2021, 250: 106540.

2. **Zhe Ding**, Junlei Shi, Qiang Gao, Qianwen Huang, Wei-Hsin Liao*. Design Sensitivity Analysis for Transient Responses of Viscoelastically Damped Systems Using Model Order Reduction Techniques. *Structural and Multidisciplinary Optimization*, https://doi.org/10.1007/s00158-021-02937-9.

3. **Zhe Ding***, Junlei Shi, Qianwen Huang, Jianyi Kong, Wei-Hsin Liao*. Sensitivity and Hessian Matrix Analysis of Power Spectrum Density Function for Non-Classically Damped Systems Subject to Nonstationary Stochastic Excitations. *Mechanical Systems and Signal Processing*, 2021, 161: 107895.

Acknowledgments

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Prof. Wei-Hsin Liao's group members

Q&A

Thank you !

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Reduction Basis I

3.1. MM method

The MM method is firstly derived from Takani-Sugeno fuzzy model which is used to represent nonlinear dynamic systems [22]. Then, Blamès [23] extended this approach to built a real-valued based projection basis formulated from the corresponding viscoelastically damped system. The projection basis of the MM method $\mathbf{T}_{MM}$ is a combination of static correction $\mathbf{X}_{cor}$ and several modal bases $\mathbf{T}_{p_j}$:

$$\mathbf{T}_{MM} = [\mathbf{X}_{cor}, \mathbf{T}_{p_1}, \cdots, \mathbf{T}_{p_m}] . \tag{22}$$

For each modal basis $\mathbf{T}_{p_j}$, it is constructed by the pseudo-normal mode solutions of the eigenvalue problem

$$\left(\mathbf{K}^*(\omega_{p_j}) - \lambda_k^{*2}(\omega_{p_j})\mathbf{M}\right) \mathbf{\Phi}_k^*(\omega_{p_j}) = \mathbf{0}, \tag{23}$$

where ω_{p_j} is a priori chosen value related to the frequency range of interest and λ_k^*, $\mathbf{\Phi}_k^*$ are the kth normal eigensolutions when the priori imposed frequency is ω_{p_j}. It is verified that good approximation results of the dynamic response of highly damped structures could be obtained when the projection bases evaluated at the minimum and the maximum frequency range of interest are included [20, 23]. If the computational

- **When ω_{p1}=0, MM is the same with MSE;**
- **Has not been applied in calculating the transient response sensitivities of viscoelastically damped systems;**
- **Influenced by ω_{pj} and the involved modes at each ω_{pj}.**

Reduction Basis II

3.2. MSER method

The MSER method aims to increase the accuracy of the approximation by iteratively seeking a better projection basis. The projection basis $\mathbf{T}_{MSER}$ is enriched by adding displacement residuals $\mathbf{R}_d^*$ to the $\mathbf{T}_{MSE}$:

$$\mathbf{T}_{MSER} = [\mathbf{T}_{MSE}, \mathbf{R}_d^*] \,. \qquad (24)$$

The displacement residuals are derived from the static response to load residuals $\mathbf{R}_f^*$:

$$\mathbf{R}_f^*(\omega) = \left(\mathbf{K}^*(\omega) - \omega^2\mathbf{M}\right)\mathbf{X}_r^*(\omega) - \mathbf{F}, \qquad (25)$$

where $\mathbf{X}_r^*(\omega)$ is the approximation of the dynamic response calculated by using the projection basis $\mathbf{T}_{MSER}$ (the initial projection basis is $\mathbf{T}_{MSE}$ and the robustness of the updated projection basis increases as the displacement residuals are added). The displacement residuals are obtained by using of the static stiffness matrix $\mathbf{K}_0 = \mathbf{K}^*\,(\omega = 0)$ and the load residuals $\mathbf{R}_f^*$, which are given by

$$\mathbf{R}_d^*(\omega) = \mathbf{K}_0^{-1}\mathbf{R}_f^*(\omega). \qquad (26)$$

$$\varepsilon_R = \frac{\left\|\mathbf{R}_d^{*H}\mathbf{K}_0\mathbf{R}_d^*\right\|_2}{\left\|\mathbf{X}_r^{*H}\mathbf{K}_0\mathbf{X}_r^*\right\|_2} < \varepsilon_{tol},$$

where ε_R is the error estimate of the displacement residuals.

- **Convergence**
- **Influenced by initial projection basis (*Nm*) and the tolerance ε_{tol}**

Reduction Basis III

3.3. IAM

The projection basis $\mathbf{T}_{IAM}$ is enriched by complex modes, which aims to reduce the error of the modal truncation problem for viscoelastically damped systems. The projection matrix $\mathbf{T}_{IAM}$ is built on a modified MSE base and three perturbation bases ($\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$) [21]

$$\mathbf{T}_{IAM} = [\mathbf{T}_{MMSE}, \mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3], \tag{28}$$

where the modal projection basis $\mathbf{T}_{MMSE}$ is defined as

$$\mathbf{T}_{MMSE} = \left[\mathbf{X}_{cor}, \mathbf{\Phi}_1(\omega_{p_j}), \cdots, \mathbf{\Phi}_n(\omega_{p_j})\right]. \tag{29}$$

$$\mathbf{T}_1 = \sum_{j=1}^{L} \frac{\varphi_j^T \mathbf{F}(s) \varphi_j}{(s-\lambda_j)\theta_j} \qquad \mathbf{T}_2 = \sum_{j=1}^{L} \frac{\varphi_j^T \mathbf{F}(s) \varphi_j}{\lambda_j \theta_j} + \mathbf{K}^{-1}\mathbf{F}(s) \quad \mathbf{T}_3 = \sum_{j=1}^{L} \frac{\varphi_j^T \mathbf{F}(s) \varphi_j}{\lambda_j^2 \theta_j} - \mathbf{K}^{-1}\mathbf{G}_0\mathbf{K}^{-1}\mathbf{F}(s), \tag{30}$$

where $\theta_j = \varphi_j^T \left(2\lambda_j \mathbf{M} + \mathbf{G}(\lambda_j) + \lambda_j \left.\frac{\partial \mathbf{G}(s)}{\partial s}\right|_{s=\lambda_j}\right) \varphi_j$ and $\mathbf{G}(s) = \sum_{k=1}^{n} G_k(s)\mathbf{C}_k$.

- **Influenced by ω_{pj} and the involved modes at each ω_{pj}**
- **Computational Time?**